LETTERS TO THE EDITOR

The Momentum of the Photon

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A recent discussion on the rest mass of the photon by Greenberg and Greenberg[1] and subsequent criticisms by Keswani[2] and Lawson[3] have arrived at conclusions very different from that of Tangherlini.[4] These differences may be traced directly to their assumed answers to the question, "How does the momentum of a photon change as it passes from vacuum into an optically dense medium?" These discussions have largely ignored observed optical phenomena such as the experiment of Jones and Richards.[5] We wish to clarify the nature of the ambiguity in answering this question.

Following the Greenbergs, we consider a beam of photons of energy density $Nh\nu$, where N is the number of photons per unit volume. The Poynting vector S for such a beam in vacuum is

$$S = Nh\nu c \tag{1}$$

and represents the energy crossing a unit area in unit time. We now assume normal incidence of the beam upon a dielectric with index of refraction n and further assume this medium to be nonabsorbing and nondispersive. We then denote the corresponding quantities in the dielectric by primes and have

$$S' = N'h\nu v, \tag{2}$$

where $v = c/n$ is the velocity of propagation in the dielectric. Under the nonessential assumption of total transmission of the photon beam across the interface, we have

$$S = S' \tag{3}$$

and find that the photon densities are related by

$$N' = Nn. \tag{4}$$

We next investigate the momentum densities G and G'. Clearly these are related to the photon momenta p and p' by

$$G = Np,$$
$$G' = N'p' = Nnp'. \tag{5}$$

The ambiguity arises when we seek to relate G and G' through their relation to the Poynting vectors we have equated in (3). This relationship between G' and S' involves knowledge of the correct form of the energy-momentum tensor which has long been a source of controversy. An extensive review and criticism of the various proposals may be found in the recent work of Brevik.[6] The nature of the disagreement may be most readily seen by consideration of the Abraham tensor which results in

$$G = S/c^2,$$
$$G' = S'/c^2, \tag{6}$$

the combination of (3), (5), and (6) then yielding

$$p' = p/n = h/\lambda' n^2. \tag{7}$$

By contrast, adoption of the Minkowski tensor results in

$$G = S/c^2,$$
$$G' = S'/v^2 = S'n^2/c^2, \tag{8}$$

the combination of (3), (5), and (8) then yielding

$$p' = np = h/\lambda'. \tag{9}$$

The contradiction between predictions (7) and (9) is great, and it might be thought that verification of one or the other would be trivial. We turn then to experiment to see if we can find how the momentum of a photon changes when it passes from vacuum to dielectric.

The experiment which might be expected to bear most directly on this question is that of Jones and Richards.[5] They made a very careful measurement of the torque delivered to a metal vane suspended in a variety of dielectric liquids. They determined to within a 1% accuracy that the pressure on the vane when in the liquid exceeded that when in air by a factor equal to the index of refraction. This would support prediction (9) which follows from Minkowski's tensor under the assumption that the entire pressure felt by the vane was due to radiation pressure. However, Brevik[6] shows that the Jones and Richards experiment cannot invalidate the Abraham tensor which results in prediction (7). The Abraham tensor demands that the photon beam be accompanied by a mechanical force density transported by the liquid and just sufficient to result in the observed pressure felt by the vane. Related observations of Snell's law and of the angle of Cerenkov radiation are in a similar way simply understood as single photon processes on the basis of (9). However, proper account of the mechanical momentum density with Abraham's tensor will still allow support for (7).

We would like to suggest that the single slit diffraction pattern coupled with the uncertainty principle would appear to argue directly for (9). Under conditions that the aperature size is much greater than a wavelength, it is easily observed with a laser, slit, and screen that the angular width $\Delta\theta$ of the diffraction pattern in vacuum is scaled down by a factor of n when the system is immersed in a dielectric,

$$\Delta\theta_{\mathrm{vac}}/\Delta\theta_{\mathrm{diel}}=\lambda_{\mathrm{vac}}/\lambda_{\mathrm{diel}}=n, \tag{10}$$

in accord with wave optical theory. In the usual manner we relate this angular width to the ratio of the uncertainty in the transverse photon momentum to the incident momentum,

$$\frac{\Delta\theta_{\mathrm{vac}}}{\Delta\theta_{\mathrm{diel}}}=\frac{(\Delta p/p)_{\mathrm{vac}}}{(\Delta p/p)_{\mathrm{diel}}}. \tag{11}$$

Equation (11) expresses the fact that the predominant forward scattering of photons following any interaction with the dielectric renders the single slit pattern a faithful record of the momentum distribution of the photons just beyond the slit. The uncertainty in the transverse photon momentum should, however, be a function only of the width of the slit and therefore,

$$\Delta p_{\mathrm{vac}}/\Delta p_{\mathrm{diel}}=1. \tag{12}$$

Combining (10)–(12) yields

$$\Delta\theta_{\mathrm{vac}}/\Delta\theta_{\mathrm{diel}}=p_{\mathrm{diel}}/p_{\mathrm{vac}}=n \tag{13}$$

which is identical with (9).

We thus suggest that the momentum of the photon scales directly with the index of refraction of a dielectric. This suggests an extended range of validity for the deBroglie relation. Thus, when a photon crosses an interface from air to water, its momentum is increased and it is slowed down. This is, of course, very different behavior from that of nonrelativistic particles where we are accustomed to the proportionality of velocity and momentum. As additional support of the divorce of momentum and velocity for photons, we note that one liquid used by Jones and Richards was carbon disulfide in which the group velocity and phase velocity differ by 3%. They claim the experimental evidence to be in favor of the mean phase velocity as the significant factor in determining the momentum of the radiation field.

[1] J. M. Greenberg and J. L. Greenberg, Amer. J. Phys. **36,** 274 (1968).
[2] G. H. Keswani, Amer. J. Phys. **39,** 231 (1971).
[3] J. D. Lawson, Amer. J. Phys. **39,** 1411 (1971).
[4] F. R. Tangherlini, Amer. J. Phys. **36,** 1001 (1968).
[5] R. V. Jones and J. C. S. Richards, Proc. Roy. Soc. (London) **221A,** 480 (1954).
[6] I. Brevik, Mat. Fys. Medd. Dan. Vid. Selsk. **37,** 11, 13 (1970).

A Note on: "Deflection of Projectiles due to Rotation of the Earth"

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Burns,[1] in a recent article in this Journal,[1] solves the vector differential equation for a projectile near the Earth's surface under two sets of concurrent conditions:

(i) general initial velocity conditions, constant gravitational acceleration, and negligible air resistance;
(ii) the velocities due to the Coriolis accelerations are neglected in the computation of the Coriolis accelerations.

Although condition (ii) is a very attractive one to impose because it simplifies the differential equations of motion, it is not consistent with the conditions under which the equations were derived. More specifically, the terms neglected are of the same order—linear in the magnitude of the angular velocity $\boldsymbol{\omega}$—as the terms which are retained. The following analysis shows that it is not necessary to impose the second set of conditions.

The equation of motion of a projectile near the Earth's surface under conditions (i), neglecting terms of $O(\omega^2)$, relative to an observer on the Earth's surface is

$$d^2\mathbf{r}/dt^2=2(d\mathbf{r}/dt)\times\boldsymbol{\omega}-\mathbf{g}.$$

Choosing a system of rectangular Cartesian coordinates x, y, z such that z is in the direction of $-\mathbf{g}$ and $\boldsymbol{\omega}$ is in the x, z plane (x pointing north) then at a point of latitude λ ($-\frac{1}{2}\pi\leq\lambda\leq\frac{1}{2}\pi$) the equations of motion are